

A MATHEMATICAL MODEL FOR TYPE II INSTRUMENTAL DISCRIMINATIONS

An Abstract of a Thesis by
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Using the same assumptions found in the Trabasso & Bower (1968) mathematical model for type I and relevant redundant cue (RRC) discriminations, a mathematical model was proposed for type II discriminations. A type I discrimination has one relevant cue. An RRC discrimination has two relevant cues, either of which could be used, independently, to solve the discrimination problem. A type II discrimination has two relevant cues, both of which must be used before the discrimination problem can be solved.

In the Trabasso & Bower (1968) model, the difficulty of a type I discrimination problem was defined in terms of the salience of the relevant cue, where salience was represented as a measure of the probability of *S* attending to a particular cue in a stimulus complex. The difficulty of an RRC discrimination problem was defined as the sum of the two relevant saliences. Since a type II discrimination requires attention and usage of both relevant cues in the problem, the difficulty of the discrimination problem was assumed to be a multiplicative function of the two relevant cue saliences.

This hypothesis was tested and supported. The mathematical model could predict the mean errors for a type II discrimination. However, the experiment also produced data which conflicted with Trabasso & Bower's (1968) assumptions. For example, *Ss* were capable of learning a new solution to a discrimination problem, even though a previously learned solution was still correct. It was concluded that changes in the underlying assumptions of the Trabasso & Bower (1968) model may necessitate a change in the mathematical model itself.

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INSTRUMENTAL DISCRIMINATIONS

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Introduction and Review of the Literature

Hull first popularized the idea of stating learning theories in mathematical language (Hull, Hovland, Ross, Hall, Perkins, & Fitch, 1940). Learning theory, at least in the several simple paradigms then being studied, had already advanced to such a state that only mathematics could state the hypotheses with enough precision to allow adequate testability. Also, the mathematical language had none of the connotations of ordinary language. Thus, mathematics was able to do away with much of the ambiguity common in natural language (Brown, 1960). Shortly after Hull initiated the work on mathematical learning models, Spence (1936) wrote "The Nature of Discrimination Learning in Animals." This was the first attempt to define discrimination problem learning using mathematical language.

The need for a mathematical model of discrimination learning was clearly seen. However, Spence's specific formulation was not uniformly accepted. The continuity-noncontinuity controversy raged for several years following the publication of Spence's paper. Spence hypothesized that discrimination responding was the result of two processes. The gradual strengthening of responses in the presence of the reinforced cue caused small increments in excitation. The gradual weakening of the responses in the presence of

the non-reinforced cue caused small increments in inhibition. Increments in excitation or inhibition were associated with the entire stimulus complex. Animals did not learn to solve discrimination problems in "all or none" fashion by attending to selected aspects of the stimulus complex (Sutherland & Mackintosh, 1971).

Today, discrimination learning is often not viewed as a gradual process (Rock, 1957). "All or none" learning models are now more prevalent as a means of describing the learning process involved with discrimination problems (Lashley, 1942). All or none learning is adequately presented by a Markov two-state model, in which, on a certain trial, the probability of *S* responding correctly to a stimulus changes from chance to certainty. Before that trial, the probability of *S* responding correctly was chance. After that trial, *S* will always respond appropriately in that discrimination problem.

Restle (1957) formulated one of the first all or none models in discrimination learning. One key assumption in his model consisted of a mandatory single cue solution. An *S* who solved a discrimination problem was assumed to attend to only one dimension of the stimulus complex, and was basing his response solely on that dimension. The data on discrimination learning does not support this contention. First, several types of solvable discrimination problems

require attention to several stimulus dimensions for their solution. For example, in Shepard, Hovland, & Jenkins' (1961) analysis of discrimination learning, six types of discrimination problems were specified. Of those six, only the type I discrimination problem could be solved by attending to a single stimulus dimension. In addition, some discrimination problems have several dimensions which can be used to solve the problem. According to Restle's single cue concept, an *S* should not be able to learn about more than one of these dimensions. However, recent research has indicated that some proportion of the total number of *Ss* are able to make the appropriate discriminative response in the presence of each of the relevant dimensions (Trabasso & Bower, 1968).

The role of mediating responses has also changed radically since the first discrimination mathematical model. Spence (1936) did not accept the notion of hypothesis testing in discrimination learning, although he might have admitted the inclusion of mediating responses into his model by way of fractional anticipatory goal responses (Hull, 1935). At this time, the concept of hypothesis testing is included in several discrimination learning theories.

Probably the model in which hypothesis testing is most explicitly formulated is the Sutherland-Mackintosh attention theory (Mackintosh, 1962). In this theory, before

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learning to respond correctly to the stimulus complex can be accomplished, *S* must learn to attend to the correct stimulus dimension. Although attention may have behavior counterparts (Mandler & Hooper, 1967), it is clearly a cognitive concept, and thus alien to Spence's formulation.

Trabasso & Bower (1968) formulated a model that contained all or none learning, hypothesis testing, and multi-look stimulus sampling. Their mathematical model of stimulus selection hypothesized that an *S* learns to solve a discrimination problem through a combination of two means, a "search mode," and a "test mode." The search mode is initiated both at the start of the discrimination task and after *S* receives an error feedback signal. Each stimulus varies along several dimensions. For instance, a sound must have at least two dimensions, pitch and frequency. In the search mode *S* first examines the stimulus and selects which cue dimension or dimensions upon which to attend. Although *S* is capable of attending to several stimulus dimensions simultaneously, the number of dimensions to which *S* is attending may vary. Sampling of stimulus dimensions is done with replacement. If a stimulus dimension is sampled more than once, that stimulus dimension is more influential in dictating the choice response than if the stimulus dimension is only sampled once. The probability that an *S* will attend to a particular dimension in a stimulus complex is

determined by the salience of that stimulus dimension.

For example, if the stimuli were cards on which a figure was drawn, the stimulus dimensions could be size, shape, and color of the figure. It is assumed that *S* first would examine the card, and then attend to one or more stimulus dimensions. Following this, *S* would formulate a hypothesis in accordance with the stimulus dimensions to which he has attended. If *S* is attending to the color dimension, and notices that the card is black, he may hypothesize that all black cues belong in the category "A," whereas all white cues belong in the category "B." The *S* will then respond "A" in the presence of black. Having formulated a hypothesis and made a response, *S* then switches to the test mode.

In the test mode, *S* tests the correctness of his hypothesis. Until *S* receives an error feedback signal, *S* will classify each subsequent stimulus card according to the dimensions on which he has focused, apply his hypothesis, and make the appropriate response. If, on any subsequent trial, *S* receives an error feedback, he immediately will revert back to the search mode. For the next stimulus card, *S* will examine the card, attend to a new sample of stimulus dimensions, and formulate a new hypothesis based on those new dimensions. The *S*, therefore, will keep alternating between search mode and test mode until the task is

completed. The task may be terminated after a set number of trials, or after S has reached a set criterion of consecutive correct responses.

In a complex stimulus discrimination task, attention to different stimulus dimensions is not independent (Trabasso & Bower, 1968). Salience of one particular stimulus dimension depends upon the noticeability of that dimension in relation to the noticeability of all other dimensions in the stimulus pattern. Noticeability is affected by innate, or species specific differences, stimulus-bound, or cue intensity variables, and past training factors (Trabasso & Bower, 1968). According to Trabasso & Bower, cue salience (a_i) can be defined by the following formula:

$$a_i = \frac{w_i}{\sum_{j=1}^N w_j} \quad (\text{formula 1})$$

Where w_i denotes the noticeability of the cue dimension i ,

and $\sum_{j=1}^N w_j$ denotes the total noticeability of all cue

dimensions in the stimulus pattern.

It follows that the more salient a stimulus dimension is, the more likely that it will be used in a hypothesis. If that dimension happens to be relevant, or must be attended to before S can formulate the correct classification

rule, then the discrimination is easier to solve. If that dimension is irrelevant, then the discrimination is more difficult to solve. Thus, learning rate is assumed to correlate with the salience of the different stimulus dimensions.

For a discrimination with only one relevant cue, an estimate of the learning rate, i.e., the probability of sampling the relevant cue, is:

$$a_i = \frac{P_i}{E(T_i) - (1 - P_i)} \quad (\text{formula 2})$$

Where $E(T_i)$ denotes the average number of errors for all Ss who solve the discrimination, a_i is the estimate of cue salience for dimension i , and P_i is the proportion of Ss who solve the discrimination.

In some discrimination problems, there is more than one relevant dimension. The addition of a relevant redundant cue (RRC) makes the discrimination problem easier to solve (Warren, 1953). Now S may solve the problem by basing his responses either on the first relevant dimension only, the relevant redundant dimension only, or both stimulus dimensions. For example, say the relevant dimensions were shape and color. Red triangle would always be "A," and blue circle would always be "B." Circle is never red, and triangle is never blue. The S may solve this discrimination by responding on the basis of either color, shape, or both

dimensions.

According to Trabasso & Bower (1968), the learning rate for the RRC problem is:

$$a_c + s = \frac{w_s + w_c}{N} \quad (\text{formula 3})$$

$$j = \sum_{j=1}^J w_j$$

Where $a_c + s$ denotes the learning rate for the RRC problem, w_s is the noticeability of the shape cue, w_c is the noticeability of the color cue, and N is the total

$$j = \sum_{j=1}^J w_j$$

noticeability for all the cues in the stimulus pattern.





Trabasso & Bower (1968) tested their mathematical model, using the RRC paradigm. First, two groups of control *SS* ($N = 45$ per group) were tested on a single, one dimension relevant discrimination problem. For example, one group of *SS* learned a discrimination problem where dot position was relevant and shape of figure was irrelevant. A second group learned the identical discrimination problem with shape of figure relevant and dot position irrelevant. From these error rate functions, Trabasso & Bower were able to predict the average number of errors required for an RRC problem consisting of both dot and geometric shape as relevant dimensions.

Ninety *SS* were tested using the RRC problem. The

estimated sampling probability for the RRC group, computed from formula 2, was .239. Formula 2 was also used to compute the estimated sampling probabilities for the two control groups. The estimated sampling probability for the dot position relevant group was .164. The estimated sampling probability of the shape of figure relevant group was .094. As predicted by the model, the sum of the estimated sampling probabilities for the two control groups, $.164 + .094 = .258$, was not significantly different from the estimated sampling probability for the RRC group. A likelihood ratio test for equality of the observed and the predicted estimates showed that differences between a_{RRC} and $a_s + a_d$ were not significant ($\chi^2 = 0.62$, $df = 1$).

In the present experiment, this model was expanded to include the type II discrimination problem. In the type II discrimination problem, there are two relevant stimulus dimensions, as in the RRC problem. However, these dimensions are not redundant. Both dimensions must be considered on each trial in order to solve the discrimination problem. If an S attends to one relevant stimulus dimension without consideration of the other relevant stimulus dimension, his response rate is still chance.

For example, the type II discrimination problem used in the present experiment had two relevant dimensions, geometric shape and dot position. Shape was either triangle

or circle. Dot was either above or below the geometric figure. In this example, cues  and  may indicate response "A," and cues  and  may indicate response "B." Here both dot position and geometric shape dimensions must be used in order to make the correct response. Classification of the various types of instrumental discriminations was done by Shepard, Hovland & Jenkins (1961).

The hypothesis being tested in the present experiment was that the rate of learning for a type II discrimination problem is equal to the product of the rates of learning for both relevant cues alone. That is:

$$a_{sd} = (a_s) (a_d) \quad (\text{formula 4})$$

Where a_{sd} is the learning rate when both dot and shape are relevant but not redundant, a_s is the learning rate when shape is relevant with dot and color irrelevant, and a_d is the learning rate when dot is relevant with shape and color irrelevant.

Learning rate for a type I discrimination problem correlates directly with the probability of the relevant cue being sampled. A type II discrimination has two relevant cues which must be sampled simultaneously. Since sampling is done with replacement, the fact that one relevant cue has been sampled will not effect the probability that the other

relevant cue will be sampled. Thus, the learning rate for a type II discrimination problem should correlate directly with the product of the two relevant cue saliences.

Following from formula 1, a cue salience is always greater than or equal to zero, and less than or equal to one ($0 < a_i < 1$). Therefore, the product of two cue saliences is always less than or equal to either of the two cue saliences alone. Since learning rate correlates with cue salience, a type II discrimination problem should be more difficult than a type I discrimination problem. Data from the Shepard, Hovland & Jenkins (1961) experiment confirm this conclusion.

Taken from formula 2, an estimate of the learning rate for a type II discrimination problem (a_{sd}) is:

$$a_{sd} = \frac{P_{sd}}{E(T_{sd}) - (1 - P_{sd})} \quad (\text{formula 5})$$

Where P_{sd} is the proportion of s who solve the discrimination problem, and $E(T_{sd})$ is the average number of errors for all Ss solving the problem. If the assumptions used to formulate the hypothesis prove correct, then:

$$(a_s)(a_d) = \frac{P_{sd}}{E(T_{sd}) - (1 - P_{sd})} \quad (\text{formula 6})$$

Where a_s is the learning rate for a type I discrimination with shape of figure as the relevant dimension, and a_d is

the learning rate for a type I discrimination with dot position as the relevant dimension. Learning rates for type I discriminations are computed by means of formula 2.

After each *S* had completed either the type II, the geometric shape relevant only, or the dot position relevant only discrimination problem, each *S* was trained on an RRC problem using both geometric shape and dot position as relevant dimensions. Sutherland & Holgate (1966) showed that after pretraining on a single dimension relevant discrimination problem, *S* will be very likely to learn and use that same dimension, rather than another dimension, or both dimensions, in a subsequent RRC discrimination problem. The more salient the relevant dimension is in the single dimension relevant discrimination, the less likely *S* will learn about the other RRC in an RRC discrimination problem. This effect has been called blocking (Kamin, 1968).

Sutherland & Holgate (1966) interpreted these results as supporting a two-process model of discrimination learning. The two processes are, first, *S* learns to attend to the relevant stimulus dimension, and then, *S* establishes the appropriate choice response to that dimension. Since *S* has already learned to attend to one stimulus dimension, the possibility that he will attend to the other stimulus dimension during the subsequent RRC discrimination problem trials is considerably decreased. This model was developed jointly

by Mackintosh & Sutherland (Mackintosh, 1962).

Transition to an RRC problem from a type II discrimination problem has an added complexity. In the type II discrimination problem choice responses must be based on both geometric shape and dot position. If responses are based on geometric shape alone, or dot position alone, responses are only reinforced on fifty per cent of the trials. Since the two stimulus dimensions are not redundant, attention to only one relevant dimension on any one trial will not increase the probability that *S* makes the correct response over chance. Each stimulus dimension is associated equally often with both available responses. Thus, the transition to an RRC problem from a type II discrimination problem involves some unlearning, and should result in more errors than the transition from a single cue relevant problem to an RRC problem.

Based on the Mackintosh-Sutherland two-process model, an *S* who has received pretraining in a type II discrimination problem should be attending to both the dot position and the geometric shape dimensions. It follows that such an *S* should be more inclined to learn both relevant dimensions in a type II discrimination problem than an *S* who has received single dimension relevant discrimination pretraining or, possibly, no pretraining at all. This is one of the hypotheses to be tested in the present experiment.

Method

Subjects

Subjects were sixty undergraduate students at Drake University. Each of the *SS* was randomly assigned to one of three groups ($N = 20$); the dot position relevant group, the geometric shape relevant group, or the type II discrimination group. The age of the *SS* ranged from 18 to 26. Subjects were assigned to groups without regard for sex or age. All *SS* were tested individually.

Stimulus Materials

Copies of the stimulus cards, drawn to a 1:3 scale are included in Appendix A. The stimuli were outlined geometric figures drawn on 3 x 5 inch file cards. Colored felt pens and templates were used to draw the figures. These geometric forms were approximately one inch square in area. The dot was 0.25 inches in diameter and located 0.25 inches above or below the geometric shape.





Three decks of cards were used. The first deck included three independent stimulus dimensions--shape, color and dot. The shape was either triangle or circle, the color was either red or green, and the dot was either above or below the geometric form. This resulted in 2^3 , or eight possible patterns. The second deck included three stimulus

dimensions, but two of the stimulus dimensions were not independent. Color was independent, but dot and shape correlated exactly. This resulted in 2^2 , or four possible patterns. Each pattern was represented twice, for a total of eight cards. The last deck consisted of single dimension cards. In half the cards, the dot position dimension was represented, but the geometric shape dimension was neutralized by the use of a square in the place of the triangle or circle. In the other half of the cards, the geometric shape dimension was represented, but the dot position dimension was eliminated. No dots were drawn on these cards. The color dimension was represented in all cards in the third deck, but the dimension, as always, was irrelevant. The third deck consisted of 2^2 or four possible patterns and eight cards.

Procedure

Each *S* was brought into a testing room individually with only *E* present. The *S* then read a set of instructions modified from the Trabasso & Bower (1968) experiment. A copy of these instructions is included in Appendix B. The first stimulus deck was brought forth and shuffled in order to randomize the order of the cards. The *S* was then shown the top card and he identified it as either an "Alpha" or a "Beta." The response was verbal. After *S* gave his response, he was told whether or not he was correct. The *S* progressed

at his own rate through the deck of cards, with the exception that there was never more than a fifteen second exposure to any one stimulus card. Cards were reshuffled each time *S* completed the deck. This procedure continued until *S* made sixteen consecutive correct responses. If, after the ninety-sixth trial, *S* still had not started his run of sixteen consecutive correct responses, the procedure was discontinued and that *S* was considered to have failed to solve the discrimination problem.

The three groups of *Ss* were distinguished by this first phase of the experiment. Group A had to learn to discriminate between the triangle and the circle. Triangle was "Alpha." Circle was "Beta." Both the color dimension and the dot position dimension were irrelevant. Group B had to learn to discriminate between the dot above the geometric figure and the dot below the geometric figure. Both the color dimension and the geometric shape dimension were irrelevant. Group C had to learn the type II discrimination. In this discrimination,  and  were "Alpha";  and  were "Beta." Both shape and dot position were relevant, but the discrimination could not be solved without attending to both dimensions simultaneously. Responses based on either relevant dimension alone were no more accurate than chance. Color was irrelevant.

Following this first phase of the experiment, *Ss* who

reached the criterion of sixteen consecutive correct responses in a row on the original discrimination problem were given 32 trials on an RRC discrimination task. Shape and dot were the relevant redundant cues, such that either shape or dot alone were sufficient to solve the discrimination. The *S* was told whether or not his response was correct after each trial. Reinforcement contingencies remained the same as in Phase I training for the dot position relevant and the geometric shape relevant groups. Each *S* proceeded at his own pace. After the sixteenth trial on this task, all *Ss* were given a single stimulus dimension card sorting task. Each *S* was handed the third deck of cards and told to separate these cards into two piles, an Alpha pile and a Beta pile, based on the rules which *S* had learned during the first sixteen RRC discrimination trials. The *Ss* went through the third deck of cards once, and received no feedback.

Each card in the third deck had only one dimension which was relevant in the RRC discrimination task. The color dimension, which was irrelevant in the RRC discrimination task, was included in the third deck of cards, but remained irrelevant. If an *S* was able to correctly categorize cards in which only a single dimension was relevant, that *S* was considered to have learned that dimension. Each relevant dimension found in the RRC discrimination task was

presented by four stimulus cards apiece, for a total of eight stimulus cards. The *S* had to correctly categorize all four stimulus cards in a relevant dimension before he was considered to have learned that dimension.

Following the first single stimulus dimension sorting task, each *S* was given sixteen additional trials on the RRC discrimination task with feedback. After *Ss* had completed these trials they were again given the stimulus dimension sorting task using deck three. The procedure was the same as in the first stimulus sorting task. This was the final task for each *S*.

Results

The mean errors to criterion [$E(T_i)$] for the type II discrimination problem was 20.55. This differed significantly from the mean errors to criterion for the dot position relevant group, 2.95, and the geometric shape relevant group, 7.4, [$F(2.57) = 17.84, p < .01$]. Five SS failed to solve the type II discrimination problem, all subjects solved the dot position relevant discrimination problem, and one S failed to solve the geometric shape relevant discrimination problem. Figure 1 shows the difference in learning rates between the three groups.

Using formula 2, the mean errors to criterion [$E(T_i)$] was converted into an estimate of the learning rates (a_i) for each group. The learning rate for the type II discrimination was .0369, a_s was .1293, and a_d was .3390. A likelihood-ratio test (Bower & Trabasso, 1964) was used to test if a_{sd} was significantly different from the other two learning rates. The learning rate for the type II discrimination was significantly different from a_s , [$\chi^2(1) = 16.018, p < .001$], and from a_d , [$\chi^2(1) = 47.397, p < .001$].

According to formula 4, a_{sd} should equal the product of a_s times a_d . This hypothesis was tested. The learning rate for the shape relevant group was multiplied by the learning rate for the dot position relevant group

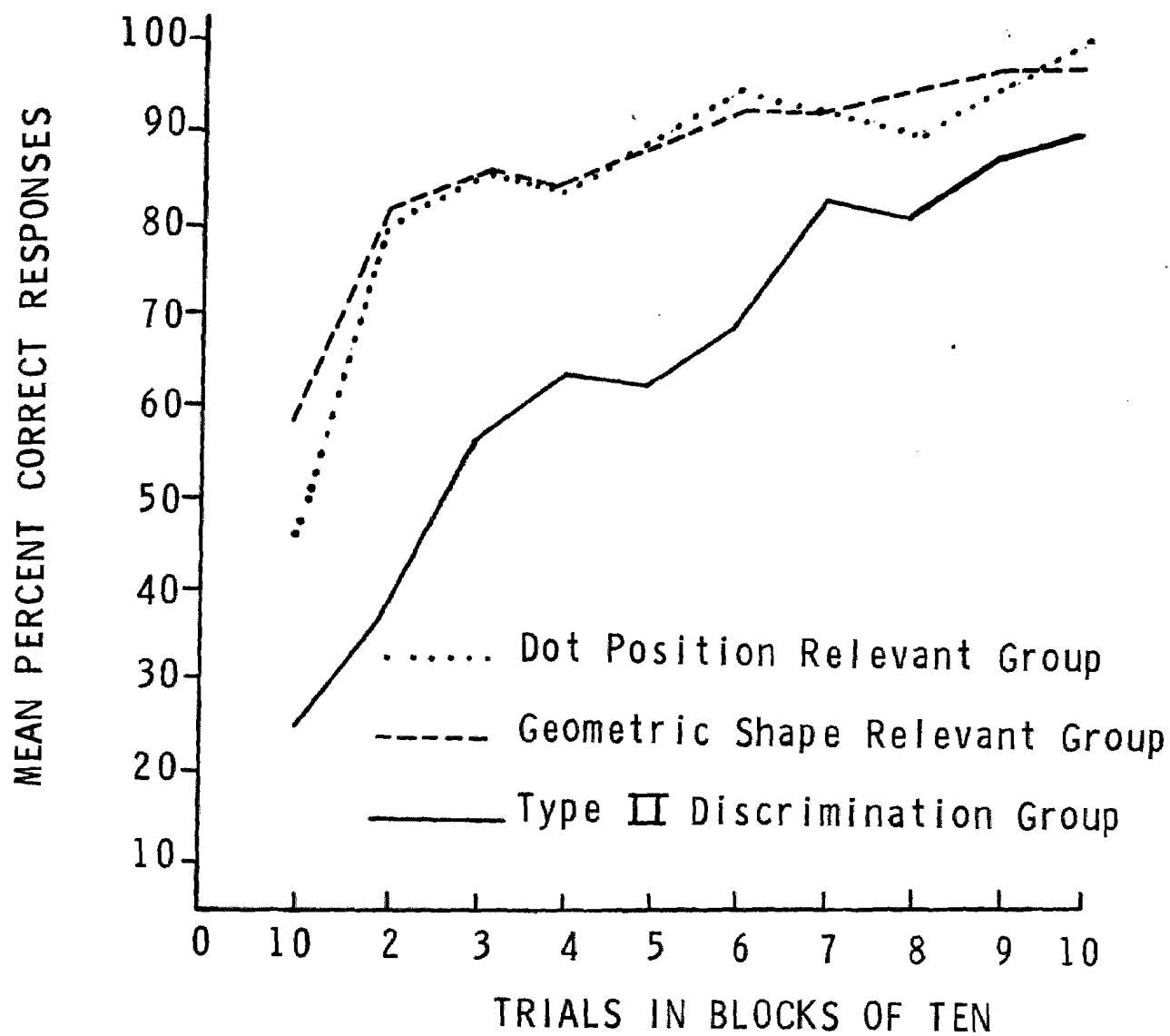


Fig. 1. Mean percent correct responses in blocks of ten trials of all groups during pretraining.

Table 2
 Acquisition Results: Mean Errors and Sampling
 Probability Estimates for all Conditions

Group	$E(T_i)$	P_i	$a(i)$	χ^2
Shape	7.40	19/20	.1293	16.018
Dot	2.95	20/20	.3390	47.397
Type II	20.55	15/20	.0339	

$$(a_s) (a_d) = .0438$$

[(.1293) (.3990)]. The product was .0438. A likelihood-ratio test showed that a_{sd} , .0369, was not significantly different from the product of a_s and a_d [$\chi^2 (1) = 0.306$, $p < .39$].

The predicted mean errors to criterion was computed from the predicted learning rate for the type II discrimination, $(a_s) (a_d)$. Formula 7, which follows from formula 2, yields $E[T_{(s) (d)}]$.

$$E[T_{(s) (d)}] = \frac{P_{(sd)}}{(a_s) (a_d)} + (1 - P_{sd}) \quad (\text{formula 7})$$

The actual proportion of S learning the type II discrimination problem, $P_{(sd)}$, was used in this calculation. The resultant $E[T_{(s) (d)}]$ was 18.06. This figure compares favorably with the actual mean errors to criterion, 20.55.

In the RRC portion of this study, more errors were predicted for the type II discrimination pretraining group than for either of the other two groups. The type II discrimination pretraining group averaged 3.27 errors on the RRC discrimination problem. An average of the errors on the RRC discrimination problem for the other two groups was .590. This difference proved significant ($t = 3$, $df = 19$, $p < .005$).

The single cue dimension stimulus cards enabled E to discover to which stimulus dimension or dimensions S had

attended during the RRC discrimination trials. It was found that pretraining, at least for the dot position group and the geometric shape group, did affect which solution was utilized in the RRC discrimination problem as evidenced by final test performance. Table 3 gives a complete breakdown of the two single stimulus dimension card sorting tasks. For the geometric shape pretraining group, eleven *SS* sorted correctly using geometric shape, but could not solve the RRC problem using dot position. Six *SS* sorted correctly using either dot position or geometric shape. One *S* was dropped from the experiment prior to testing, and two *SS* did not sort correctly using either of the cues.

For the dot position pretraining group, fourteen *SS* sorted correctly using only dot position as the relevant cue, while six *SS* sorted correctly using either geometric shape and dot position as relevant cues. A Pearson's χ^2 goodness of fit test was used to test the possibility that these results occurred by chance. These results were significantly different from chance ($\chi^2 = 16.99$, $df = 2$, $p < .001$). Thus, it appears that although pretraining on a certain stimulus dimension results in *SS* preferring that dimension on a subsequent RRC discrimination problem, some *SS* also learn the non-pretrained redundant cue.

Forty per cent of the *SS* in the type II discrimination pretraining group prior to the RRC task sorted

Table 3
 Manner in Which Subjects Solve the RRC
 Discrimination Task After 16 Trials

Pretraining Group:	Shape	Dot	Type II
Solution:			
Shape	11	0	1
Dot	0	14	3
Neither	2	0	5
Both	6	6	6
Total Subjects	19	20	15

correctly on both relevant dimensions. Only thirty per cent of the dot relevant dimension pretraining group, and thirty-two per cent of the shape relevant pretraining group learned about both relevant dimensions in the RRC discrimination task. However, these differences were not statistically significant.

Of the fifteen *Ss* who completed the type II discrimination pretraining task, six sorted correctly on both cues, three used dot position only, and one used geometric shape only. Thus, nine *Ss* could solve the RRC discrimination on the basis of the dot position cue, but only seven *Ss* could solve the RRC discrimination on the basis of the geometric position cue. Although the number of *Ss* who solved the RRC discrimination on the basis of the dot position cue was not significantly greater than the number of *Ss* who solved the discrimination on the basis of the geometric shape cue ($\chi^2 = .5129$, $df = 3$), the data was in a logical direction. The dot position cue was the more salient cue.

Each *S* received sixteen overtraining trials on the RRC task following the initial single stimulus dimension card assortment task. Subjects were then retested on the single stimulus dimension assortment task. During those overtraining trials, *Ss* from the shape and dot pretraining groups tended to learn about the least preferred stimulus dimension. Five *Ss* in the dot position pretraining group

learned about the geometric shape dimension during the over-training trials. Four *Ss* in the geometric shape pretraining group learned about the dot position dimension. Three more *Ss* in the type II discrimination pretraining group learned to respond correctly to both cues. These data are shown in Table 4.

Table 4
 Manner in Which Subjects Solve the RRC
 Discrimination Task After 32 Trials

Pretraining Group:	Shape	Dot	Type II
Solution:			
Shape	8	0	0
Dot	0	9	2
Neither	1	0	4
Both	10	11	9
Total Subjects	19	20	15

Discussion

The value of the learning rate for type II discrimination problem, a_{sd} , was predicted successfully from the learning rates of two type I discrimination problems, a_s and a_d . This finding not only serves to support and extend Trabasso & Bower's (1968) mathematical model, but also demonstrates that mathematical models can be formulated to make predictions about discriminations beyond the type I discrimination problem. Most mathematical models are incapable of formulating predictions for discriminations whose solution involves attention to anymore than one stimulus dimension. Clearly, subsequent mathematical models of discrimination learning should be designed to account for vaious types of discrimination learning paradigms. Broadening the scope of the mathematical model not only serves to generalize the model, but also gives added opportunities to test the model.

The Trabasso & Bower model was expanded and was still able to sustain an empirical challenge, however, data were generated in present experiment which cast some doubt on certain of the assumptions of the model. The Trabasso & Bower model assumes that S will only return to the search mode after S makes an error in the discrimination problem. Therefore, once an S has found a solution, no new stimulus

dimensions should be sampled, no new hypotheses should be formulated, and no new solutions should be tested. Once *S* has learned one solution to a discrimination, the previously learned solution should block any attempt at learning any additional solution.

In two instances, results from the present experiment show the improbability of this assumption. First, some *Ss* who mastered a single dimension relevant discrimination problem were able to learn to classify an RRC discrimination problem on the basis of both relevant dimensions. According to Trabasso & Bower, *S* should have been able to classify the RRC discrimination cards only on the basis of that dimension learned in the single dimension relevant discrimination problem. The first learned dimension should block attention to the second relevant dimension. However, studies by Kamin (1968) and Mackintosh (1965) have demonstrated that blocking is a graded phenomenon. That is, although something is learned about the added dimension, it is much less than that learned about the pretrained dimension.

Trabasso & Bower (1968) tried a procedure very similar to the procedure used in this experiment. The results they obtained were in conflict with the results obtained in the present experiment. Pretraining on a single dimension discrimination caused almost complete blocking of all other stimulus dimensions and made the learning of both

relevant dimensions in an RRC discrimination unlikely. The difference between these findings and the findings in the present experiment may be due to the fact that the change from the single stimulus relevant pretraining phase to the RRC phase of the experiment was much less obvious in the Trabasso & Bower procedure than in the present procedure. In the latter experiment, no attempt was made to hide the fact that a second deck of cards was being used for the RRC phase of the experiment. In the Trabasso & Bower experiment, apparently the transition to the RRC deck of cards was made without *S* being aware of it. This difference in procedure may have resulted in *Ss* in the present experiment treating the RRC problem as if it was a new and different task, while *Ss* in the Trabasso & Bower study treated the RRC problem as a continuation of the original pretraining task.

In addition, the cue saliences used in the present experiment were assumed to be greater than the cue saliences used in the Trabasso & Bower study, and may have resulted in a "reduced" blocking effect. In the Trabasso & Bower experiment, five different stimulus dimensions were used. In the present experiment, only three different stimulus dimensions were used. This difference may have resulted in Trabasso & Bower's *S* having a stronger attention response built up to the relevant pretraining stimulus dimension prior to entering RRC training than *Ss* in the present

experiment. The number of different stimulus dimensions used in the present experiment was only three because a type II discrimination task is considerably more difficult to solve than a type I or RRC task. Pilot research had indicated that with five stimulus dimensions, too few *Ss* were able to solve the type II problem within the allotted number of trials.

A second instance in which Trabasso & Bower's assumption is faulty is found in the final phases of the present study, the two testing problems. The assumption predicts that the same number of *Ss* should be able to respond correctly using either of the RRC's in the second test as in the first. Actually, it was here demonstrated that a greater number of *Ss* were able to respond correctly using either of the RRC's in the second test session than in the first. This occurred despite the absence of errors in the second half of the RRC task; a condition considered necessary by Trabasso & Bower's model for resampling of cues. Thus, it appears that, at least, some *Ss* resampled cues and learned about the novel cue appearing only in the RRC task. Although some research has previously indicated that *Ss* may learn about the less preferred cue in RRC training with extended RRC training (Pavlov, 1927; Sutherland & Holgate, 1966), this has not been demonstrated to occur in the blocking paradigm. One possible explanation for the finding in





the present experiment may be that with cue saliences of sufficient strength, stimulus sampling does occur with extended training in the RRC paradigm.

Another finding that warrants discussion, but was correctly hypothesized, was that type II discrimination pretraining prior to RRC training retarded performance on the RRC task. Subjects from the type II discrimination pretraining group made more errors, and needed more trials, in learning the RRC problem than *Ss* from either of the two type I discrimination pretraining groups. Possibly, *Ss* from the former group approached the RRC problem as if it was a novel problem. As a result of type II discrimination pretraining, an *S's* responses must be based upon values of both relevant dimensions used in that problem. However, neither single dimension alone is sufficient to solve the problem. Upon entering the RRC phase of training, *S* may use one or both of the previously relevant dimensions in order to respond correctly. But, the specific cue-response attachments reinforced during pretraining are not always reinforced during RRC training. By virtue of the nature of the type II pretraining task itself, only half of the responses based upon a single relevant dimension are reinforced. This means that when RRC training begins, half of *S's* responses will be reinforced if he is still employing the strategy learned during type II discrimination pretraining. The result is an

increase in errors during RRC training. This does not occur for *Ss* pretrained on a type I discrimination task prior to RRC training because responses based upon the pretraining dimension continue to be reinforced one hundred per cent of the time during RRC training.

The performance of type II discrimination pretrained *Ss* during the RRC training also appears to relate to final test performance. A greater percentage of these *Ss* learned about both cues in the RRC task, as evidenced by test performances, than did type I pretrained *Ss*. Presumably, this occurred because the salience of the cues used in the RRC task was more nearly alike at the beginning of the RRC task for type II pretrained *Ss* than for type I pretrained *Ss*. The latter *S* came into the RRC task using a highly successful and, therefore, salient, pretraining cue which remained a perfectly reliable predictor of the correct response during RRC training. According to the Trabasso & Bower model, the making of correct responses by attending to the pretraining cue during RRC training both limits the probability of sampling other cues in the RRC task and therefore decreases their salience. It follows that a blocking effect should be indicated during subsequent testing. For the type II discrimination pretrained *Ss*, not only were the saliences of the two relevant cues employed in the RRC task at similar levels at the commencement of the RRC task, but the nature

of Type II discrimination pretraining promoted the occurrence of errors during RRC training in order to solve that task. Either or both of these conditions could have resulted in the finding that a greater percentage of type II discrimination pretrained *Ss* learned about both cues in the RRC task than did type I discrimination pretrained *Ss* (Bower and Trabasso, 1964).

A final point should also be noted with regard to type II discrimination pretraining performance. Instead of behaving as if these *Ss* were working in a Markov two-process system, many of them learned the problem as if they were working in a Markov three-process system. The two-process system assumes only a change from chance performance to perfect or one hundred per cent performance. In the present experiment, the type II discrimination pretrained *Ss* went from chance level responding to seventy-five per cent probability of responding correctly to errorless responding. The way in which they achieved this is depicted in Figure 2. For example, suppose the type II discrimination problem was  = alpha,  = beta,  = beta, and  = alpha. A typical *S* may have initially started this problem by responding at chance level to each of the four stimuli, or by responding solely on the basis of shape or dot position. This, too, results in chance level responding. The next step may have been that *S* learns to respond alpha to

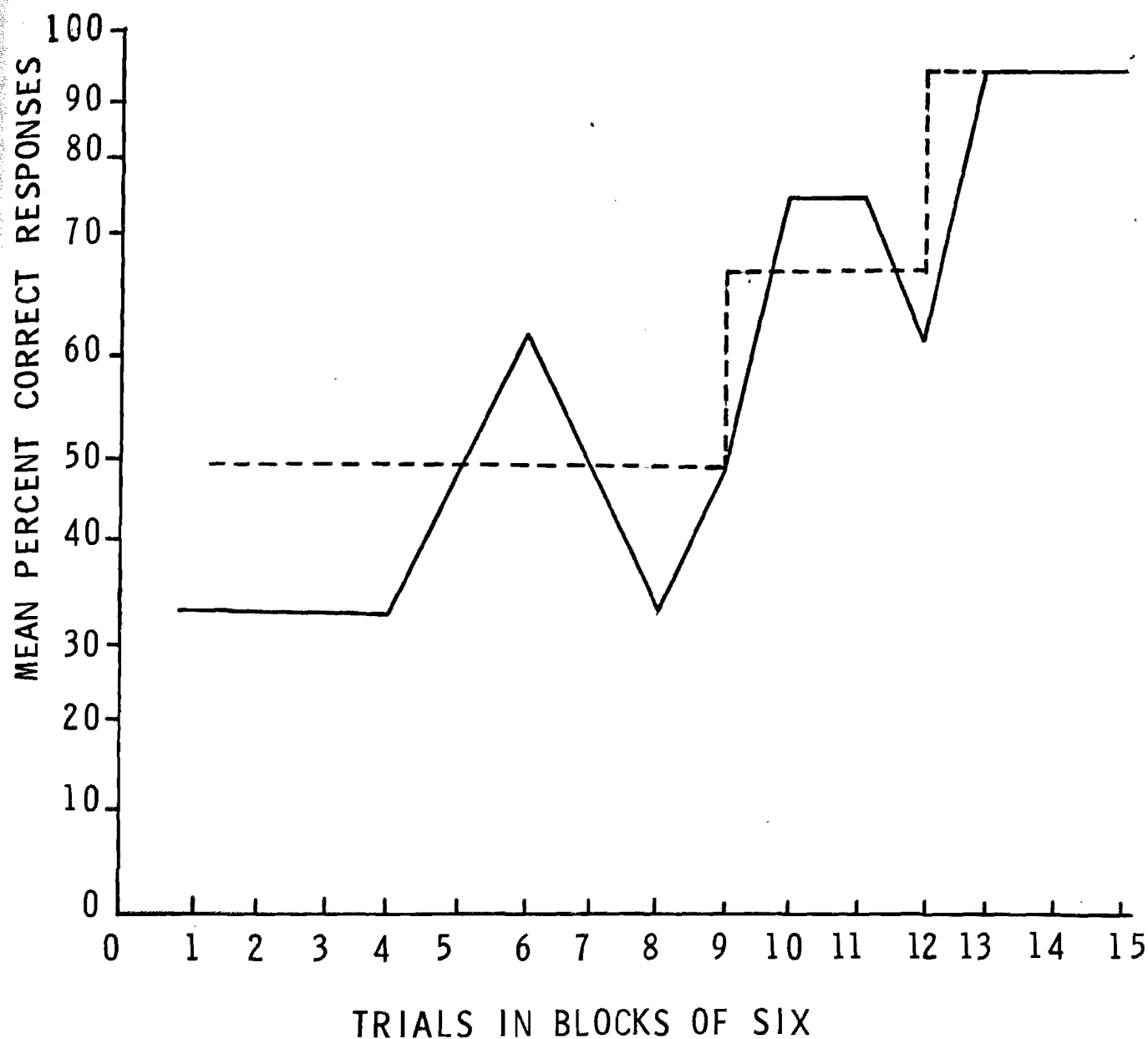




Fig. 2 Mean number correct responses in blocks of six trials by s 19 during pretraining. Solid line indicates theoretical predicted performance according to a Markov three-stage model.

 and beta to  , but at chance when the stimulus is a circle. This results in a performance that is seventy-five per cent correct. Finally, *S* may learn to respond correctly in the presence of each of the two circle stimuli. This results in errorless, or one hundred per cent correct, performance. Thus, unlike type I discrimination problems, for type II discrimination problems the "all or none" process is probably not completely adequate to account for performance.

It has been shown that a successful model for type II discrimination can be formulated. Within limits, this model was successful. However, any future mathematical model of discrimination learning may have to use different assumptions than the Trabasso & Bower (1968) model. First, the new model should account for possible resampling of the stimulus set, even after the discrimination has been learned. And, second, the new model must modify the "all or none" process assumption when analyzing higher-order discrimination problems, like the type II task.

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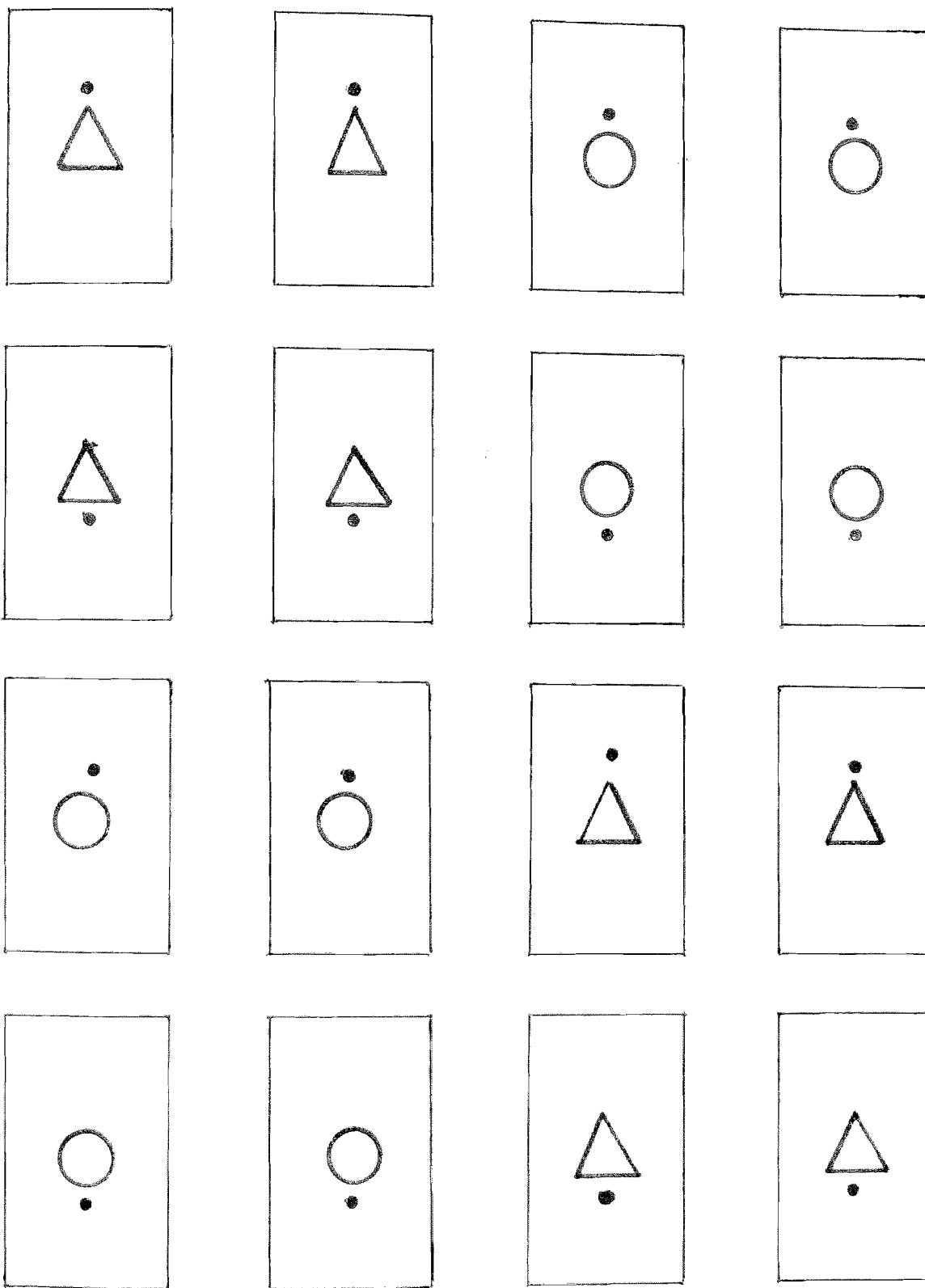
APPENDIX A

Copies of the Stimulus Cards

Deck One: Pretraining
Deck Two: RRCs
Deck Three: Single Stimulus
 Dimensions

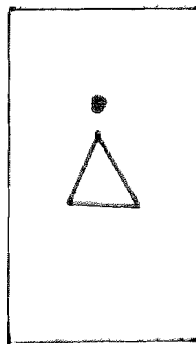
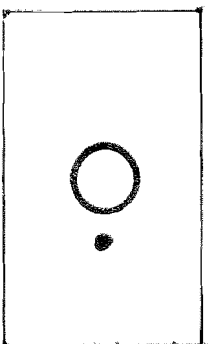
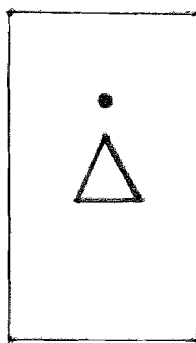
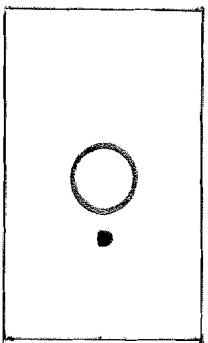
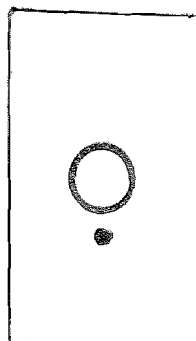
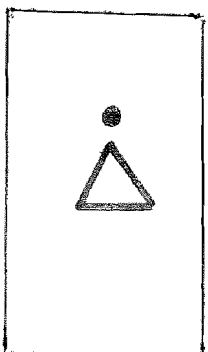
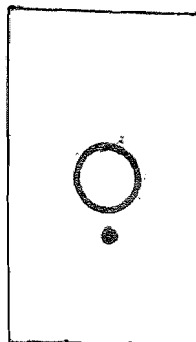
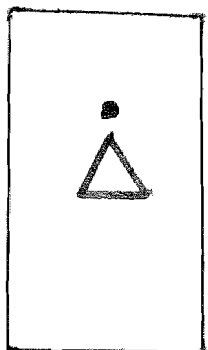
Deck One: Pretraining

scale: 1:3



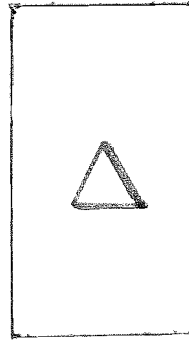
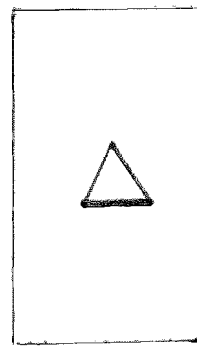
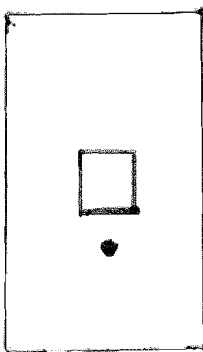
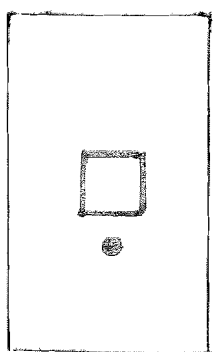
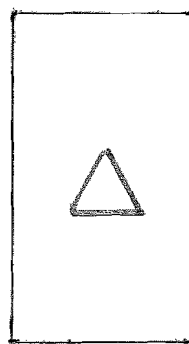
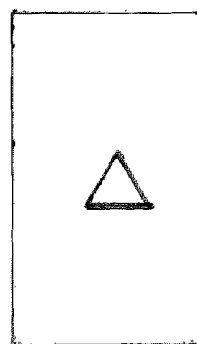
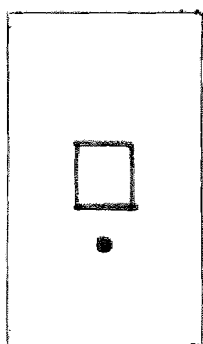
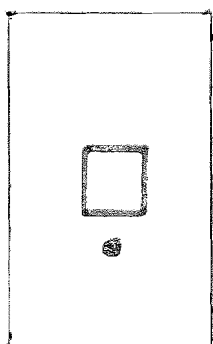
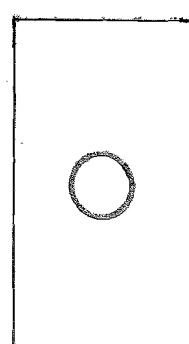
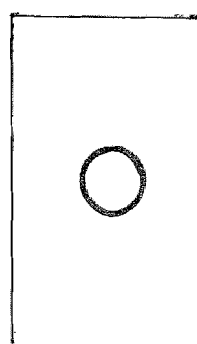
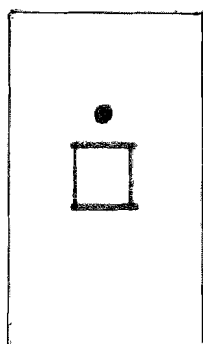
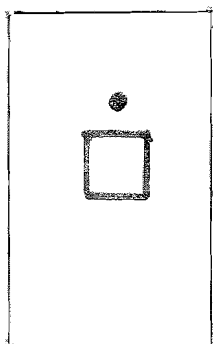
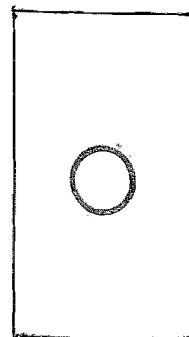
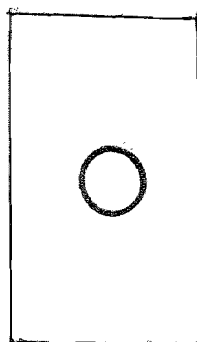
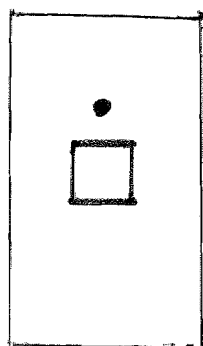
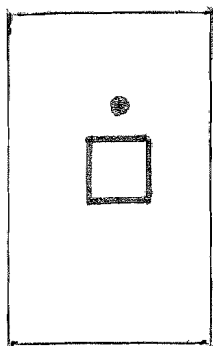
Deck Two: RRCs

scale 1:3



Deck Three: Single Stimulus Dimensions

scale 1:3



APPENDIX B

Instructions to the Subject

Appendix B

Instructions to the Subject

The purpose of this experiment is to find out how college students learn to make classifications. I have a deck of cards which may be divided into two classes, called Alpha and Beta. Each card belongs to only one category. Your job is to learn in which category a card belongs. I will show you one card at a time, and you are to classify the card as either an Alpha or a Beta. At first you must guess the category since you do not know the classification. After you classify the card, I will show you the correct answer. Then you will have a few seconds to study the card. I will then show you the next card to be classified. After awhile, you should learn a rule which will enable you to classify every card correctly as an Alpha or a Beta.

Before we begin, let me familiarize you with the nature of the cards. Here are two examples of cards which differ in several ways. The cards may differ in terms of (1) the shape of the figure, either a circle or a triangle; (2) the position of the dot, either above or below the figure; and (3) the color, either red or green.

The classification of the card will depend only on what appears on the card and nothing else. The cards are

shuffled so that the order of the cards is not important. To review, I will show you one card at a time and you are to classify it as an Alpha or a Beta. I will show you the correct classification and then we shall go on to the next card. Guess on the first card. You can learn to classify the cards by a rule. Be accurate and avoid careless mistakes.

Appendix B

Table 1

Errors to Criterion on the Pretraining Task

Subject	Group:	Shape	Dot	Type II
1.		0	0	3
2.		1	0	3
3.		1	0	7
4.		1	0	7
5.		1	0	8
6.		1	0	10
7.		2	1	12
8.		2	1	19
9.		2	1	19
10.		2	1	21
11.		3	2	22
12.		4	2	23
13.		5	2	23*
14.		5	2	28*
15.		7	3	29
16.		10	3	29
17.		12	4	29
18.		21	5	34*
19.		27	13	40
20.		41*	19	45

* Subjects who did not reach criterion.

Appendix B

Table 5

Errors in the RRC Discrimination Task

Subject	Group:	Shape	Dot	Type II
1.		0	0	0
2.		0	0	0
3.		0	0	0
4.		0	0	1
5.		0	0	1
6.		0	0	1
7.		0	0	3
8.		0	0	3
9.		0	0	3
10.		0	0	3
11.		0	0	4
12.		0	0	6
13.		0	0	6
14.		0	1	7
15.		1	1	10
16.		1	1	
17.		1	1	
18.		2	1	
19.		2	1	
20.			5	